MATH 2E Prep: Partial derivatives

Facts to Know:

1. Rules for finding partial derivatives of $u = f(x, y, z)$
   - Notation: $f_x(x, y, z) = f_x = \frac{\partial u}{\partial x} = \frac{\partial f}{\partial x} = \frac{\partial}{\partial x} f(x, y, z)$
   - To find f_x, regard y, z as constants and differentiate $f(x, y, z)$ with respect to x.
   - To find f_y, regard x, z as constants and differentiate $f(x, y, z)$ with respect to y.
   - Similar for f_z.
2. Definition of gradient:
   - $f(x, y, z)$ function of 3 variables, $\nabla f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle$

Examples:

1. Calculate the gradient of the function $f(x, y, z) = x + ye^z$, and its value at two points $(1, 0, 2)$ and $(1, 1, 0)$.

$\frac{\partial f}{\partial x} = 1$, $\frac{\partial f}{\partial y} = 0 + 1 \cdot e^z$, $\frac{\partial f}{\partial z} = 0 + ye^z$

$\Rightarrow \nabla f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle = \langle 1, e^z, ye^z \rangle$

$(\nabla f)(1, 0, 2) = \langle 1, e^2, 0 \cdot e^2 \rangle = \langle 1, e^2, 0 \rangle$

$(\nabla f)(1, 1, 0) = \langle 1, e^0, 1 \cdot e^0 \rangle = \langle 1, 1, 1 \rangle$